Building word problems: What does it take?

By Angela T. Barlow
Our “flat” world requires the instructional skill to create authentic mathematical word problems relating to real-time issues that students wrestle with in their daily lives. An emphasis on teaching through problem solving necessitates a problem-creation framework. I have made problem solving a focus this year with a class of third graders that I teach. Each lesson is filmed for possible use in our elementary school math methods courses at the university. After watching a video clip from a recent lesson, teacher candidates wanted to know where I had obtained the problem that the students solved. When I replied that I created it, the reaction was, “Oh, I could never do that.”

As I reflected on this response, I thought, Why not? Isn’t creating mathematical word problems a skill that teachers should have? Answering this question is twofold.

**Why problem solve?**

First, why should a teacher focus on engaging students in problem solving? In my classroom, we know we are engaged in problem solving when our response to reading a problem is, “Huh?” We talk about this idea that problem solving involves a step where you may not immediately know how to proceed. These conversations are important because many students enter my classroom believing that such problems are “too hard” or that they are “not smart enough” to do them. By engaging in problem solving, students begin to believe that the problems are challenging, as opposed to “too hard.” Our interpretations of problem solving align with definitions in the literature (Hiebert et al. 1997; Lambdin 2003; NCTM 2000), and students begin to see that they are “smart enough.”

Just as important as improved dispositions, though, is the development of mathematical understanding. Teaching through problem solving offers students opportunities to connect what they are learning to familiar contexts as well as to previously learned material. This, in turn, leads to initial understanding. With understanding comes the motivation to learn, deeper understanding, and the ability to transfer knowledge to new situations (Lambdin 2003). In other words, for learning mathematics, problem solving is a vehicle, not an end goal: “Good problems give students the chance to solidify and extend what they know and, when well chosen, can stimulate mathematics learning” (NCTM 2000, p. 51).
Teachers must anticipate and make available any tools that students might need.

Why create my own word problems?
The second question is, Why not simply rely on the curriculum materials or other available resources? In recognition of the role that problem solving plays in the mathematics classroom, many resources furnish problems appropriate for use in elementary school classrooms. Resource books often supply problems organized around various problem-solving strategies (see, for example, O’Connell 2000). Additionally, curriculum materials typically associated with classroom textbooks are already in our classrooms and include problems appropriate for use in our lessons. With the variety of resources that are available, why would a teacher need to be able to create mathematical word problems?

The skill of creating problems empowers teachers to tailor instruction to meet their students’ learning needs. Teachers can focus on developing problems that pinpoint a particular concept or objective in their lessons. Alternatively, teachers can design problems that force their students to confront currently held misconceptions or areas of difficulty. By creating our own problems, we can explicitly link the math to students’ experiences and cultures. Problems of this nature are often absent or misplaced in our curriculum materials. With the recognition of the limitations of our curriculum materials (Russell et al. 2003) comes the recognition that the ability to create appropriate problems is an important skill that teachers should develop.

The World Is Flat (Friedman 2005) describes the globalization that advances in technology have imposed on the world economy in recent decades. For our country to maintain its stature in the world, future citizens must be prepared to problem solve and apply their skills to new situations. These future citizens are sitting in our elementary school classrooms today. The skill of creating meaningful, effective mathematical word problems is more than an important teacher skill; it is an essential teacher skill for preparing students to function in this flat world. Consequently, one must ask, What skills are involved in creating a problem? Which thought process or processes lead to the creation of a mathematically appropriate problem?

Frameworks exist for solving problems as well as for judging the quality of problems (see, for example, Pólya 1945 and Breyfogle and Williams 2008–2009, respectively). However, no established frameworks are currently available to describe the process of creating problems. Given that the ability to create authentic mathematical problems is an essential teacher skill in this flat world, having such a framework is crucial.

What is the process?
The framework to create mathematical word problems consists of five basic steps:

1. Identify the mathematical goals.
2. Decide on a problem context.
3. Create the problem.
4. Anticipate students’ solutions.
5. Implement and reflect on the problem.

Presented in this way, the process seems straightforward. The example that follows, however,
shows that this is not necessarily the case. Before examining the sample problem, consider a third-grade classroom episode.

What is the context?
In this classroom, the terms unit, long, flat, and large cube refer to base-ten blocks. In previous lessons, students had established that if the value of a unit is 1, the values of the long, the flat, and the large cubes are 10, 100, and 1000, respectively. (Other textbooks or curriculums may not use these terms in this manner.)

The teacher began by reading the word problem aloud:

Let me read this, and let’s see if we all understand what the problem is asking: “On Thursday, Alexis was at home decomposing numbers with base-ten blocks. The value of her blocks”—which means, you know, what they were worth—the ones she was working with, “was...” Andrea, can you read that number for me?

“Two thousand, forty-three.”

OK. “Two thousand, forty-three. When she wasn’t looking, her little brother grabbed a flat and two longs. What is the value of Alexis’ remaining blocks?” Now, I want to—for you all to—think about what it is that you know in this problem. What are some things we know? Alexandrea, tell me what’s one thing you know in this problem?

“There’s not any flats.”

“Hmm?”

“There’s not any flats.”

“You notice there are no flats. OK. Kesha, what’s something you know from the problem?”

“Well, her brother took two longs.”

“Her brother took two longs. What else did he take?”

“And one flat.”

“He took a flat, too. What’s something else that we know? Frankie?”

“It’s a subtracting problem.”

“You say it’s a subtracting problem. What’s something else that we know from the problem, Jasmine?”

“She’s decomposing numbers with the base-ten blocks.”

“OK. So, she’s decomposing numbers. She’s using the base-ten blocks. Mary Ann, what’s something else we know?”

“That she has a little brother.”

“She has a little brother. What else is so very important in this that we have to use to solve the problem that we haven’t said?”

Latoya answers the teacher, “Use pictures or words to describe how you solved the problem.”

“Yeah, that’s what we are gonna do. Jim, what’s something else we know?”

“That if they take—if her little brother took—two of the longs, then there would be two longs left.”

“OK. Don’t start solving it. But what’s the other very important piece of information that we need?”

[Students mumble.] “The number she started with. OK. That was the initial value.”

[Students begin solving the problem. The following summarizes ideas proposed by students either in group work or in presentations.]

- **Alexandrea and Kesha** said that it was impossible for the little brother to take a flat because there were no flats. Therefore, he took only two longs, yielding the answer 2023 (see fig. 1). Alexandrea’s journal entry described her thinking.
- **Josh and Jermaine** stated that Alexis had extra blocks she was unaware of, which allowed her little brother to take a flat, although Alexis supposedly did not have any. Josh later recognized that Alexis had a large cube, ten flats, four longs, and three units...
Alexandrea’s journal tells why she and Kesha both thought it was impossible for the little brother to take a flat.

If Alexis’s brother took a flat then there are no flats at all because at first Alexis had none. flats, so now Alexis still doesn’t have any. flats. And if Alexis’s brother took two longs Alexis would have two longs left. So now her number is 2,023. We know because you don’t have to add the digits you half to subtract the digits.

Josh and Jermaine initially stated that Alexis must have been unaware of extra blocks she had. Pretending to cut off one flat from a large cube, Josh later realized a scenario that made it possible for the little brother to take a flat and two longs. His journal entry describes the idea of ten flats representing one large cube.

She made a big cube with a lot of flats and the brother took one of the big cube then took the longs.

Two students used the traditional subtraction algorithm to solve the problem correctly. Paige’s journal gives an explanation for the same question that had stumped Frankie.

<table>
<thead>
<tr>
<th>10</th>
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<tbody>
<tr>
<td>2,043</td>
</tr>
<tr>
<td>-2,020</td>
</tr>
<tr>
<td>1923</td>
</tr>
</tbody>
</table>

Alexis’s total number of blocks now is 1923. I got that number by using that simple subtraction problem.

She could have had 10 flats and traded them in for 1 big cube.

What follows is a description of the creation process that led to the problem highlighted in the classroom episode above.

**Step 1: Identify the math goal**

In our curriculum, second graders work on subtracting three-digit numbers, and third graders work on subtracting four-digit numbers. The majority of students in my third-grade classroom came to me with knowledge of the traditional algorithm for subtraction. However, their understanding of this algorithm was limited; and their recall of the “steps” was filled with inaccuracies. Before this classroom episode, students had been representing the composition and decomposition of numbers with base-ten blocks. Following this work, and recognizing the inaccuracies present in their current understanding of the algorithm, I needed a problem to link the physical representation of the composition and decomposition of four-digit numbers with the mathematical ideas underlying the subtraction algorithm—the focus of upcoming lessons. Specifically, I wanted a problem that would force students to think about subtraction with regrouping. The mathematical goal was for students to represent with base-ten blocks the process of four-digit subtraction with regrouping. I also wanted to
include a zero in the minuend because of known challenges with this particular situation.

**Step 2: Decide on a problem context**

Where is this mathematics used? Once your mathematical goal has been determined, the next step is to decide on a context that will facilitate students’ engagement in the mathematical ideas associated with your goal. For the previously identified mathematical goal, I needed a context to which students could relate and which would force them to think about the process of regrouping. Developing subtraction problems is easy. Creating a problem that will engage students in the process of problem solving around the ideas of subtraction and regrouping is more challenging.

Since the mathematical goal focuses on subtraction, I had to decide which model of subtraction to use. Subtraction problems for third graders are typically either take-away or compare problems (see fig. 4). I decided to use the take-away model because it aligns with the subtraction algorithm involving regrouping. This alignment would facilitate having students act out the problem with base-ten blocks and allow students access to its mathematical ideas. I also elected to use the context of representing number decomposition, since this had been the focus of our most recent lessons. My problem would have a student represent the decomposition of numbers with base-ten blocks and have her sibling take away some blocks. In thinking about this, I noted that students would understand the context of the problem because they had represented the decomposition of numbers with base-ten blocks in previous lessons. Also, students would relate to the problem context and would easily imagine the little brother taking away the blocks.

**Step 3: Create the problem**

What, exactly, should the problem say? With the mathematics and context identified, the next step is to create the problem. In doing so, make sure that the details of the problem will facilitate students’ thinking about the mathematics at hand. Problem details to consider include the numbers. Choosing the wrong numbers may be a distraction or may even hide the important mathematics. In this problem, number selection was particularly crucial, as these numbers would establish the need to model regrouping.

I decided to use the number 2043 for several reasons. First, our state’s curriculum framework calls for a four-digit number. Second, the number contains relatively small digits (2, 4, 0, and 3), therefore requiring fewer base-ten blocks. I had previously noted that working with too many blocks at one time sometimes distracts some of my students, preventing them from focusing on the math at hand.

Third, I wanted students to attend to regrouping with a single zero in the number. The presence of two zeros in the number or even the need to regroup a second time would not have allowed everyone to focus on this one really big idea, an idea with which students often struggle. I decided to use future problems for other regrouping situations. The number 2043 would allow students to focus on the lesson’s mathematical goal, subtraction with regrouping.

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**FIGURE 4**

Third-grade subtraction problems typically model how to either take-away or compare.

**Take-away Model**

Jim had 5 toy cars. He gave 3 of his toy cars to his friend Marcus. How many toy cars does Jim have now?

2 cars left

**Compare Model**

Jim has 5 toy cars. His friend Marcus has 3 toy cars. How many more toy cars does Jim have compared to Marcus?

2 more cars
involving a zero in the minuend. The problem I created appears again below for easy reference:

On Thursday, Alexis was at home decomposing numbers with base-ten blocks. The value of her blocks was 2043. When she was not looking, her little brother grabbed a flat and two longs. What is the value of Alexis’s remaining blocks? (Use pictures and words to describe how you solved the problem.)

In creating this problem, I made the decision to state that the brother grabbed a flat and two longs as opposed to stating that the brother took blocks with a value of 120. This decision would force students to think about representing the subtraction process with the blocks. Additionally, having the little brother take two longs would allow students to think about how to model the take-away process and to successfully do so with the longs. With this done, their thought processes could focus on how to take away a flat when, in their minds, there are no flats—the mathematical point of the problem.

Step 4: Anticipate students’ solutions
With the problem written, you must now think about how students will solve the problem. Which manipulatives might they use? What strategies might they try? How will students solve the problem? Will the problem engage them in problem solving?

As I thought about the problem I had created, I anticipated that students would use the base-ten blocks, since these had been our primary tools in recent lessons. In terms of strategies, the problem lends itself toward an act-it-out approach; therefore, I felt this would be most students’ strategy. I did, however, think some students might draw pictures of the base-ten blocks as a means of solving the problem and representing their thinking. I imagined students representing 2043 with two large cubes, four longs, and three units. They would then realize that to decompose one large cube into ten flats thereby makes it possible to take away a flat and two longs. The mental struggle occurring before coming to this realization would engage students in problem solving.

In reviewing the actual solutions that my students used, I was surprised that a student would decompose both large cubes into flats. This, however, gave us an alternate strategy to examine and think about how it was alike and different from other strategies. Although I had not anticipated that students would try to establish that Alexis had additional blocks nearby that she was unaware of, I appreciated students’ creativity. I was also surprised that some students elected to ignore the fact that the brother stole one flat, stating that it was impossible. Looking back, I realize now that teachers must also anticipate students’ processes that will lead to incorrect solutions.

Step 5: Implement and reflect
Now that the lesson has been implemented, how might I change it if I use it again? Reflecting on the problem’s success (or lack of success) is important. Did the problem facilitate students’ thinking about the mathematics associated with the lesson goals? Did the problem engage students in problem solving? Were students interested in the problem context? Did they understand the problem context?

Upon reflection, I felt that the context was understandable and motivated students to
think. I had mixed feelings, though, regarding the level of engagement in problem solving for all students. That is to say, those students who thought about the issue of how it was possible for the little brother to steal a flat were truly engaged in problem solving and were thinking about the mathematics associated with my goal. On the other hand, I questioned whether those who created extra blocks or ignored the fact that a flat was taken were actually engaged in problem solving. In retrospect, I believe the problem could have successfully engaged all students in problem solving if I had asked such a question at the beginning of the lesson as, Does Alexis have extra blocks that we do not know about?

Why build on this framework?

As I look back on the process of creating the word problem, two aspects of the framework are worth noting. First are the parallels between Pólya’s problem-solving process (Pólya 1945) and the steps in the problem-creating framework (see fig. 5). As a teacher establishes the mathematical goal, he or she is working to identify students’ needs, concepts or misconceptions that must be faced, students’ experiences, and so forth. This process equates to how students understand the problem as they solve it. A problem creator’s plan must address the problem context, the components associated with the problem, and students’ anticipated solutions. Once the plan has been devised, carrying it out (implementing the problem) and looking back (reflecting on the problem) are the remaining steps. Comparing the two frameworks shows that the problem-solving process provides a foundation for thinking about the problem-creating process.

Second, the five steps associated with the problem-creating framework seem uncomplicated at first glance: Identify the mathematical goal, decide on a problem context, create the problem, anticipate students’ solutions, and implement and reflect. One does not have to look far below the surface, however, to see the complexities of creating a problem. For example, deciding to address the concept of subtraction is inadequate; I had to consider the various subtraction models. Also, this problem would have taken on a completely different focus had the problem stated that the brother took blocks with a value of 120. Students’ focus would have been on remembering the algorithm they had learned the previous year. Clearly, teachers must concentrate on the mathematical ideas when moving through the framework (see fig. 6).
What can we conclude?

Being taught through problem solving motivates students to learn and simultaneously allows for learning with understanding (Lambin 2003). Creating the word problems empowers teachers to design problems that meet the unique needs and interests of their students. Such problems may emphasize a particular concept, address mathematical concepts known to cause difficulty, or link mathematics to students’ different cultural experiences and interests. Thus, designing mathematical problems is an essential teacher skill.

On the surface, writing a word problem seems like a simple task. As the example demonstrates, this is not always the case. The task of creating a word problem can become quite overwhelming when a teacher begins to consider a mathematical topic (such as subtraction), the models associated with that topic (such as take-away), the context, the role of the numbers in the problem, and so on. The problem-creating framework serves as a means for organizing one’s thoughts around this process, making it manageable by outlining the problem aspects to consider.

In many ways, the process of creating a word problem is comparable to that of solving one. Although a problem-solving framework does not provide a “recipe” for solving problems, it does offer a means of describing the necessary process for solving problems. The same is true for the problem-creating framework. It does not supply a recipe for creating problems but instead outlines the process through which the teacher must move in order to design challenging, engaging problems.

The framework not only provides structure to creating problems with the intent of teaching through problem solving but also facilitates creating a problem that is specifically designed to match the needs of the students in a class. The subtraction problem worked well in my class because it met the needs and backgrounds of those particular students as well as the mathematical goals of the lesson. The same problem may not be as appropriate for other third graders with different backgrounds and interests. However, other teachers could use the same problem-creating framework to generate problems specific to their classrooms and objectives. With the aid of the framework and a little practice, teacher candidates and practicing teachers can become proficient at designing high-quality problems—an essential skill for effectively instructing competent problem solvers in our rapidly advancing flat world.

REFERENCES


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