Many experts point to algebra as a great barrier to further learning in mathematics. For many children, however, the difficulties begin years before they encounter a formal algebra course, and one major stumbling block is the study of fractions. The research community has offered numerous reasons for difficulties in learning fractions, suggesting that the material is being taught too abstractly, too procedurally, and outside any meaningful contexts (National Research Council 2001). Furthermore, some of the problems observed in students’ understanding of fractions suggest that difficulties stem from learning fractions through rote memorization of procedures without a connection “between what is taught and their informal ways and means of operating” (Steffe and Olive 2002, p. 128).

This article describes the experiences of two teachers who sought to address deficiencies in students’ understanding of fractions. The unit on fractions described here was an attempt to provide students with a hands-on approach that fosters fraction sense, addresses common misconceptions, and provides a learning environment that develops conceptual understanding (Van de Walle and Lovin 2006). One of the primary goals was to create opportunities for students to be “actively involved with the subject matter rather than being passive receivers of information” (Gunter, Estes, and Schwab 2003, p. 273).

Christina Silvas-Centeno, a middle school teacher, wanted to enhance her education in mathematics and consequently enrolled in university mathematics classes offered by Cheryl Roddick, a university professor. In these classes, designed for teachers of grades K–8, Silvas-Centeno learned about research-based methods for teaching mathematics to children. In particular, she became familiar with constructivist pedagogy and the idea that a teacher should guide students in exploring ideas and concepts in order to create their own knowledge (Marsh and Willis 2003). She also gained understanding about encouraging student-generated strategies to help children make sense of mathematics problems (Empson 2002). One course Silvas-Centeno took was a problem-solving course for middle school teachers, where she learned how to implement problem solving throughout the mathematics curriculum. These classes served as a catalyst for changing her teaching approach in the classroom, and she began to completely rethink her
approach to teaching mathematics.

As Silvas-Centeno progressed through university classes and implemented research-based strategies in her own classroom, she noted observable changes in her students’ understanding. Yet a continuing frustration was that her sixth-grade students did not understand fractions. They did not seem to retain an understanding of fractions from previous grades, and her classroom instruction did not seem to be effective in addressing this deficiency.

Together, we discussed the problems her students were having with fractions. We came to the conclusion that she was presenting fractions in a manner that was too abstract and too focused on procedures and that her students could benefit from a more hands-on and constructivist approach.

Silvas-Centeno had learned about this approach in her university mathematics classes for teachers and agreed that manipulatives would be helpful for her sixth-grade students in creating meaning while learning fractions. Manipulatives would enable her to spend more time on the concept of fractions as well as developing meaning for the procedures.

The Unit on Fractions

Our approach to fractions involves conceptual understanding through a hands-on approach. Pattern blocks are used as the foundation for understanding all aspects of fraction concepts as well as computations. The idea of fair trade, a theme throughout the lessons in the unit, is used to create equivalent fractions. This idea enhances conceptual understanding and helps give meaning to the procedures related to fraction operations. Real-life problems also add context to the operations. Students are given word problems involving fractions and are asked to make sense of the word problems by using pattern blocks and creating models. Van de Walle and Lovin’s work (2006) was a great reference and source of inspiration during the development and implementation of the activities.
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Each group of pattern blocks needs 2 blue rhombuses to represent a value of 2/3, and out of the 12 rhombuses 6 such groups can be made. The use of equivalent fractions, orchestrated by a fair trade, determines the following pattern: $4 \div \frac{2}{3} = \frac{12}{3} \div \frac{2}{3} = 6$. Retracing the steps involved when manipulating the pattern blocks also makes it clear that the answer (6) was obtained by multiplying 4 by 3 and then dividing by 2. Observing these patterns can help students understand that this is the same answer obtained when they apply the algorithm, and yet the fair trade approach connects better to their informal ways of operating. This process also allows the use of equivalent fractions to essentially reduce this problem to one involving whole numbers: $12 \div 2 = 6$.

**Planning the Lessons**

The unit on fractions was developed with input from Roddick and Silvas-Centeno and presented in a team-teaching format. Two differing perspectives—that of the researcher and that of the teacher—were useful at each stage in the development of the lessons. The researcher contributed her mathematical understanding of the fraction concepts as well as understanding of the connections between and among other mathematical topics. The teacher contributed her perspective on state and district standards and her experience with students’ mathematical development at the sixth-grade level.

During the planning phase, much time was spent discussing the students’ needs and the lessons’ appropriateness in meeting those needs as well as addressing inconsistencies and misunderstandings. For example, many students have not developed number sense about the relative size of different fractions. Using pattern blocks should provide experiences for them to relate a fraction’s size to its corresponding numeric representation and to compare fractions (i.e., 1/2 is greater than 1/3 because the pattern block representing 1/2 is bigger than the pattern block representing 1/3). Further, some students also cannot remember the algorithms for fraction arithmetic and get confused about when common denominators are needed. Pattern blocks and fair trades can aid the students in making sense of real-world problems involving addition, subtraction, multiplication, and division of fractions. The actions performed with the pattern blocks can then be connected to the algorithms of fraction arithmetic. Much attention was given to these issues when designing the lessons.
Implementation and Refinement

During the six weeks that we taught this unit, the students participated in many cooperative-learning sessions while using pattern blocks. They were given numerous opportunities to present their work on the board and explain their understandings and solutions to the class. Although we had agreed to allow whatever amount of time was necessary for each lesson, we were continually surprised. One of the first lessons, the King’s Crown problem (see fig. 2), which we had estimated would take two days, actually took an entire week. We both understood, however, that this activity and the discussions surrounding the lesson held many key concepts that would be used throughout the unit: the ideas of fair trade, understanding the size of common fractions, and determining equivalent fractions. Thus, the additional time devoted to the lesson was worthwhile.

In the King’s Crown problem, two key fractions must be determined: 1/2 and 1/3. Students can find 1/2 of the king’s crown by trading in the red trapezoid shape for three green triangles, so that the crown is then made up of six triangles. Half of the six triangles would be three triangles. Similarly, one-third of the six triangles can be found by trading for triangles and also by observing that three blue rhombuses cover the crown completely (see fig. 3). Thus, 1/3 can be represented by either 2 triangles or 1 rhombus.

During this lesson, we first experienced the continuous cycle of frustration and elation. For example, one issue that arose was finding 1/3 of 6 objects. Students were able to find 1/3 of 3 objects, but they had difficulty finding 1/3 of a group of objects that was a multiple of three. At this point, we realized that students were viewing 1/3 as “one of three things,” not one of three equal groups making up a whole. It also became clear to us that students were not making the connection to division and were unable to see that finding 1/3 of 6 is the same as dividing 6 into 3 equal groups. These observations were quite frustrating to us, although we conceded that this lack of understanding might have resulted from a largely algorithmic approach to fractions presented to students in previous grades.

It took at least a week for the students to feel comfortable using the pattern blocks to help them solve problems involving fractions. At the end of the first week, we experienced a sense of elation as students began to explain mathematics conceptually, using the pattern blocks and the term fair trade in their explanations.

We had made several assumptions regarding what prior understanding of fractions the students would bring to the lessons. These assumptions led to numerous adjustments in the lessons, primarily a need to reintroduce basic concepts. One important discovery was that the majority of students did not understand the relationship between the numerator and denominator with respect to part to whole. Another misconception related to the notion of creating pieces of equal sizes. For example, many students believed that one-third was represented by one of three pieces and that the three pieces were not necessarily equal. Once these misconceptions were exposed, the subsequent lessons were altered to establish these ideas within the context of learning other fractional concepts.

Another unexpected factor arose as students tried to solve problems for homework. Without the assistance of their teacher or peers, they would revert to applying algorithms that they partially remembered from fifth grade. Well-meaning parents and other relatives often reinforced the focus on procedures, and thus homework results often proved frustrating. The next day, we would try to recover from their procedural mistakes and return the focus to the concepts and the pattern blocks.
One surprising result of this unit, however, was related to the students’ ability to solve word problems. According to Empson (2002), students at a young age have an informal understanding of different strategies for solving problems. Throughout this unit, the students were encouraged to use student-generated strategies (Empson 2002), and by attempting to make sense of their mathematics calculations in a meaningful way, they simultaneously improved their problem-solving skills. Furthermore, they actually gained confidence as well as improving their ability to solve word problems correctly. The students’ increased confidence in problem solving was a surprising outcome of this unit.

**Impact of This Experience on Teaching and Learning in the Classroom**

This project represented a nontraditional approach to teaching fractions, and the students struggled with the lessons in the beginning. It became clear that the students had had little experience with concept-based learning and were unaccustomed to using manipulatives to learn about fractions. Such activities as explaining their solution methods to their classmates, finding patterns, and connecting new ideas to prior mathematics were also new to them. Through this unit, the students encountered an environment that promoted constructing mathematical ideas instead of using algorithms to solve problems. It was a slow start because the students were uncomfortable learning in such an unfamiliar way. But as their confidence increased, the lessons proceeded at a much faster rate. Even though the unit started painstakingly, the students rose to the desired expectations, and mathematical growth was evident in them all. The students began to make sense of pattern block models, a step that led to a deeper understanding of fractions as well as an understanding of the algorithms for fraction arithmetic.

**Benefits for Teacher Professional Development**

In this section, we focus on the perspective of the teacher, and the voice used here reflects her voice:

This collaborative event was beneficial to my students and my teaching career. The process of planning lessons with a mentor and joining forces in the classroom was a great opportunity for professional growth. During their careers, all teachers should have an opportunity to participate in a similar event. First, lesson planning becomes a negotiated practice: both parties evaluate lessons, and student assessment is viewed from an additional source. Second, my perspective and understanding of fractions has evolved and developed. This process has encouraged me to look at concepts from different points of view, and, as a result, my understanding of fractions has reached a higher level. A teacher’s practice becomes more skillful and her perspective is altered in a way that would not have been possible without such an experience.

**Concluding Remarks**

Although not all these sixth-grade students achieved an understanding of fractions at the level we intended, they all demonstrated mathematical growth. Higher-achieving students were able to make the connections between pattern block manipulations and associated pencil-and-paper manipulations. The students already proficient in the algorithms showed an increase in their conceptual understanding of fractions. Although the lower-achieving students experienced more difficulties in making the intended connections, these students had more deficiencies in their knowledge that had to be addressed. Clearly, these students could have benefited more from this
hands-on approach to teaching fractions had they experienced it before sixth grade.

Similar lessons can be implemented and manipulatives used to enhance conceptual understanding in the classroom at any grade. Mathematics lessons that involve manipulatives allow students to develop reasoning skills, explore mathematical connections, and build number sense. Elementary school teachers can prepare their students for greater abstraction of concepts by offering concrete problem-solving scenarios. Students who are offered rich problem-solving opportunities develop a deeper understanding of concepts and are able to apply their understanding to their future learning.

Bibliography


